The exam consists of 25 questions worth 4 points each. There are 13 total pages.

**Useful Equations:**

\[ \Psi_n(x) = \left( \frac{2}{L} \right)^{1/2} \sin\left( \frac{n\pi x}{L} \right), \quad E_n = \frac{n^2 \hbar^2}{8mL^2} \]

\[ p = \frac{\hbar}{i} \frac{d}{dx} \]

\[ E_I = \frac{l(l+1)\hbar^2}{2I} \]
Point total

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Problems:

(24 points)
1. Below is the potential energy for a particle in an infinite box. The wavefunction and energy expression are given on the cover sheet.

a. Write down the Hamiltonian for this system. Give the explicit functional forms for the operators.
b. Sketch on the diagram the first two energy levels.

c. Sketch on the diagram the wavefunction for the ground state.

d. Sketch on the diagram the probability density for finding the particle as a function of x for the first excited state.
e. Evaluate the energy difference between the $n=3$ and $n=4$ levels.

f. For the ground state, give the integral expression for the probability of finding the particle between $x=L/2$ and $x=L$. Give the probability or evaluate the integral.
(16 points)
2. Below is a sketch of the harmonic oscillator potential with frequency $\nu$.

a. Draw the energy levels for the ground state ($v=0$), the first excited state ($v=1$), and the second excited state ($v=2$). Clearly mark the values of the energies.

b. Identify the zero point energy on the graph and give its value.
c. Set up the expression for the finding the particle inside the classically allowed region for the ground state. Use $\Psi_0$ to designate the wave function. Be sure to give the limits of integration. You do not have to evaluate the integral.

d. Set up the integral expression for the average value of the momentum $<p>$ for the ground state. Include the explicit functional form of the operator. You can assume that the wave function is normalized. Give the value of the integral.
(12 points)
3. For the particle on a sphere (or rotating diatomic molecule or rotating rigid rotor) with moment of inertia, I, consider the rotational state with quantum number $\ell=4$.

   a. What is the energy of the state?

   b. What is the length of the angular momentum vector?

   c. What is the degeneracy of this energy state?
Einstein discovered the photoelectric effect. The metal sodium has a work function of 2.30 eV.

a. Make a plot of KE of emitted electron vs energy ($h\nu = \hbar \omega$) of the incident light. Use energy units of eV.

b. If the incident light has $h\nu = \hbar \omega = 2.31$ eV, what is the kinetic energy of the emitted electron?
(8 points)

5. For the particle in an infinite box, the wavefunctions are given on the cover sheet.

   a. Write down one integral that is equal to one because the wavefunctions are normalized.

   b. Write down one integral that is equal to zero because the wavefunctions are orthogonal.
1. An important experimental confirmation of the de Broglie relation was provided by
   a. Measurements of the intensity patterns of electrons scattered from a solid surface.
   b. Measurements of the energy of black-body radiation as a function of wavelength.
   c. Measurements of the energies of photoelectrons emitted by metals versus wavelength of incident light.
   d. Measurements of the low temperature heat capacities of solids.
   e. All the above.

2. For the operator \(-3 \frac{d}{dx}\)
   a. \(\exp(3x)\) is an eigenfunction having eigenvalue 3.
   b. \(\exp(2x)\) is an eigenfunction having eigenvalue 2.
   c. \(\exp(-2x)\) is an eigenfunction having eigenvalue -2.
   d. \(\exp(-5x)\) is an eigenfunction having eigenvalue -5.
   e. All of the above is a true statement.

3. Which statement is true? Planck’s assumption that the oscillators in the walls of a black-body radiator have quantized energies
   a. explained the line spectra in black-body emissions.
   b. overcame the ultraviolet catastrophe.
   c. predicted a frequency of maximum power output that is independent of temperature.
   d. required the assumption that different metals have different work functions.
   e. None of the above is a true statement.
4. It is found that a particle in a one-dimensional “box” of length \( L \) can be excited from the \( n = 1 \) to the \( n = 2 \) state by light of frequency \( \nu \). If the “box” length is halved, the frequency needed to produce the \( n = 1 \) to the \( n = 2 \) transition becomes

a. \( \nu / 4 \)
b. \( \nu / 2 \)
c. \( 2 \nu \)
d. \( 4 \nu \)
e. None of the above is correct.

5. Which one of the following is a **false** statement for the particle in a one-dimensional “box” with infinite walls at \( x = 0 \) and \( L \)?

a. When \( n = 2 \), the probability for finding the particle on one side of the “box” is positive, and the probability for finding it on the other side is negative.
b. When \( n = 1 \), the probability distribution for the particle is uniform (i.e., constant from \( x = 0 \) to \( x = L \)).
c. As \( n \) increases, the energy levels get closer and closer together.
d. As \( L \) increases, the energy levels get farther apart.
e. All of the above statements are false.

6. Given below are values of the total spin quantum number, \( S \), for a many-electron atom along with the associated values of \( M_S \). Which set of values is complete and correct?

a. \( S=2, M_S=0,1,2 \)
b. \( S=1, M_S=-1,0,1 \)
c. \( S=0, M_S=-1,0,1 \)
d. \( S=-1, M_S=-1,0,1 \)
e. \( S=1, M_S=-1/2, 1/2 \)
7. Which one of the following statements **conflicts** with the quantum mechanical results for a particle undergoing a one-dimensional harmonic oscillation?

   a. The lighter the mass of the particle, the greater will be its vibrational zero-point energy.
   b. The vibrational frequency of a quantum oscillator is the same as that of a classical oscillator.
   c. Increasing the vibrational force constant increases the spacing between successive energy levels.
   d. The spacing between successive energy levels increases as the vibrational quantum number is increased.

8. Light of wavelength $4.33 \times 10^{-6}$ m excites a harmonic oscillator from its ground to its first excited vibrational state. The wavelength of light that would accomplish this same excitation if the oscillator mass were doubled but no other change occurred is

   a. $6.12 \times 10^{-6}$ m
   b. $8.66 \times 10^{-6}$ m
   c. $3.06 \times 10^{-6}$ m
   d. $4.33 \times 10^{-6}$ m
   e. $2.16 \times 10^{-6}$ m