\[ g_I = \frac{2 \pi \hbar}{M_N B_0} = \frac{\hbar}{M_N B_0} \]

\[ g_I = \frac{(6.626 \times 10^{-34} \text{Js})(107 \times 10^6 \text{s}^{-1})}{(5.051 \times 10^{-27} \text{J/T})(10 \text{T})} = 1.404 \]

ii. From Table 15.2

\(^{13}\text{C}\) has \(g\)-Factor = 1.405

1.b) i. For spin-\(\frac{1}{2}\) nuclei, \(\Delta E = g \times \hbar B_0\)

\[ \Delta E = (26.75 \times 10^7 \text{T}^{-1} \text{s}^{-1}) \left( \frac{6.626 \times 10^{-34} \text{Js}}{2 \pi} \right) (20 \text{T}) \]

\[ \Delta E = 5.64 \times 10^{-25} \text{J} \]

\[ \Delta E (\text{eV}) = \frac{\Delta E}{e} = \frac{5.64 \times 10^{-25} \text{J}}{1.602 \times 10^{-19} \text{Js/eV}} = 3.52 \times 10^{-6} \text{eV} \]
1.b)iii. \[ \frac{N_B}{N_A} = e^{-\frac{\Delta E}{kT}} = e^{-\frac{(5.64 \times 10^{-25} J)}{(1.381 \times 10^{-23} J/K)(300K)}} = 0.9999 \]

1.c)i. \text{abs. intensity } \propto (N_A - N_B)B_0

\[ N_A - N_B = \frac{N_T X B_0}{2kT} \]

so \text{abs. intensity } \propto \frac{N_T X B_0}{2kT}

\begin{align*}
\text{160 MHz instrument:} & & B_0 = \frac{2\pi \nu L}{\gamma} = \frac{2\pi (160 \times 10^6 \text{Hz})}{2.673 \times 10^7 \text{T}^{-1} \text{s}^{-1}} \\
& & B_0 = 3.76 \text{T} \\
\text{850 MHz instrument:} & & B_0 = \frac{2\pi (850 \times 10^6 \text{s}^{-1})}{2.673 \times 10^7 \text{T}^{-1} \text{s}^{-1}} = 20.0 \text{T}
\end{align*}

want same absorption intensity, so equate expressions

\[ \frac{N_1 \frac{\partial \chi}{\partial T}}{2k(300k)} (3.76 T)^2 \frac{N_2 \frac{\partial \chi}{\partial T}}{2k(4.2k)} (20.0 T)^2 \]

\[ \frac{N_2}{N_1} = \frac{4.2k (3.76 T)^2}{300k (20.0 T)^2} = 0.000495 \]

\[ N_1 = 1.00 \text{ M} \]

\[ N_2 = 4.95 \times 10^{-4} \text{ M} \]

much greater sensitivity!
1.c)ii. No. \( d = \frac{\nu - \nu_0}{\nu_0} \times 10^6 \)

\( \nu = \) Larmor Freq. of nucleus in question
\( \nu_0 = \) Larmor Freq. of standard

while \( \nu \) and \( \nu_0 \) both depend on \( B_0 \) as

\[ \nu = (1 - \sigma) \frac{\gamma B_0}{2\pi}, \]

since they both have this dependence, dividing \( \nu - \nu_0 \)
by \( \nu_0 \) removes the dependence of \( \sigma \) on \( B_0 \).

The chemical shift is therefore independent of \( B_0 \)
and thus the NMR instrument.

1.c)iii. No. The separation between peaks = \( J \) which is

a constant (For a given pair of nuclei in a molecule)

which has units of Hz.

For example, if \( J = 10 \text{ Hz} \) and \( \nu_0 = 160 \text{ MHz} \), then

peaks in the triplet would be separated by

\[ \frac{10 \text{ Hz}}{160 \times 10^6 \text{ Hz}} \times 10^6 = 0.0625 \text{ ppm}. \]
1.c) iii. (continued)

But, if $v_0 = 850 \text{ MHz}$, then the peaks would be separated by only

$$\frac{10^6 \text{ Hz}}{850 \cdot 10^6 \text{ Hz}} = 0.0118 \text{ ppm}.$$

1.d) $N_\alpha - N_\beta \approx \frac{N_\alpha N_\beta}{2kT} \beta_0 = \frac{2kT(N_\alpha - N_\beta)}{N_\alpha N_\beta}$

$N_\alpha - N_\beta = 1.77 \cdot 10^{11} \text{ cm}^{-3}$

$k = 1.38 \cdot 10^{-23} \text{ J/K}$

$T = 300 \text{ K}$

$j = 6.73 \cdot 10^7 \text{ T s}^{-1}$

$\hbar = 1.055 \cdot 10^{-34} \text{ J s}$

$N = \text{ total concentration of } ^{13}\text{C atoms}$

in solution in $\text{cm}^{-3}$

$[\text{CH}_3\text{OH}] = 2.50 \cdot 10^{-3} \text{ mol/L}$

$= 2.50 \cdot 10^{-3} \text{ mol/L} \left( 6.022 \cdot 10^{23} \text{ molecules/mol} \right) \left( \frac{1.000 \text{ L}}{1000 \text{ cm}^3} \right)$

$= 1.51 \cdot 10^{18} \text{ cm}^{-3}$
1. d) (continued)

\[ [^{13}\text{C atoms}] = 1.1\% \quad [\text{CH}_3\text{OH}] = 1.66 \times 10^{16} \text{ cm}^{-3} \]

combining all terms...

\[
B_0 = \frac{2(1.381 \times 10^{-23})(300)(1.77 \times 10^n \text{ cm}^{-3})}{(1.66 \times 10^{16} \text{ cm}^{-3})(6.73 \times 10^7)(1.055 \times 10^{-34})}
\]

\[ B_0 = 12.4 \text{ T} \]

2. a) Eq. 15.19 \quad \delta = (\sigma_0 - \sigma) \times 10^6

\[ \sigma_0 = \text{shielding constant for TMS} \]

\[ \sigma_0 \text{ is large because TMS is } e^- \text{ rich} \]

\[ \delta \text{ is large only if } \sigma_0 - \sigma \text{ is large requiring that } \sigma \ll \sigma_0. \]

2. b) Yes. TMS is electron-rich which strongly shields the \(^1\text{H}\) giving rise to low \(\nu_L = (1-\sigma) \frac{\gamma B_0}{2\pi} \).

The same \(e^-\) will shield the \(^{13}\text{C}\) nucleus giving rise to a low \(\nu_L\) for this resonance.
2.c) looking at Fig. 15.11

\[
E = -m_A \hbar V_L(A) - m_x \hbar V_L(x) + m_A m_x \hbar J
\]

(note, a diagram like Fig. 15.12 is OK as long as everything is labeled).

2.d) 0.1 ppm at 500 MHz

\[
\frac{J}{5.00 \cdot 10^8 \text{Hz}} \cdot 10^6 = 0.1 \Rightarrow J = 50 \text{ Hz}
\]

\[
\text{ppm (250 MHz)} = \frac{50}{2.50 \cdot 10^8 \text{Hz}} \cdot 10^6 = 0.2
\]
2. e) Spin-lattice relaxation occurs by α and β spins returning to equilibrium distribution, 

\[ \frac{N_\beta}{N_\alpha} = e^{-\frac{h\nu_L}{kT}} \], after having been perturbed by application of a \( \frac{\pi}{2} \) or a π-pulse.

Transverse relaxation occurs as nuclei which experience different local magnetic fields, \( B_{\text{loc}} \), precess at different Larmor Frequencies, \( \nu_L = \frac{g\mu_B B_{\text{loc}}}{2\pi} \), in the xy plane after application of a \( \frac{\pi}{2} \)-pulse. The spread of spins reduces the magnetization vector picked up by the detector without necessarily changing the population of α and β spins.

\[ \frac{\pi}{2} \text{ pulse} \quad \nu_L > \langle \nu_L \rangle \]

\[ \pi \text{-pulse} \quad \nu_L < \langle \nu_L \rangle \]

\( M_x \) or \( M_z \) shrinks as α and β spins return to equilibrium (\( M \) points again along +z axis).

My shrinks as spin vectors spread out in xy plane.