1.a) recall that \( K = \frac{n\pi}{L} \). For the P. in a Box, so the \( n=0 \) state corresponds to the \( K=0 \) state which is allowed for the Free particle.

\[
|\Psi|^2 = |A e^{i(0)x} + Be^{-i(0)x}|^2
\]

\[
|\Psi|^2 = |A + B|^2
\]

1.b) \( x \) is defined over **all** space \((-\infty, \infty)\)

\( \Psi_1 = 0 \)

no curvature

\( \Psi_2 = \sqrt{\frac{\alpha}{L}} \sin\left(\frac{n\pi x}{L}\right) \)

curvature is sinusoidal

\( \Psi_3 = 0 \)

no curvature

---

Ok to say curved, concave down, wavy, et cetera.

Also Ok to calc. 2nd derivative.

Just important to recognize \( \Psi \) is defined everywhere but has different shapes in various regions.
1.c) \( P = \int_{0}^{\frac{L}{4}} |\Psi|^2 \, dx = \frac{2}{L} \int_{0}^{\frac{L}{4}} \sin^2 \left( \frac{2\pi x}{L} \right) \, dx \)

\[ \sin^2 y = \frac{1}{2} (1 - \cos 2y) \]

\[ P = \frac{1}{L} \int_{0}^{\frac{L}{4}} 1 - \cos \left( \frac{4\pi x}{L} \right) \, dx = \frac{1}{L} \left[ x - \left( \frac{L}{4\pi} \right) \sin \left( \frac{4\pi x}{L} \right) \right]_{0}^{\frac{L}{4}} \]

\[ P = \frac{1}{4} - \frac{1}{4\pi} \sin \left( \frac{\pi}{4} \right) = \frac{1}{4} \]

\[ \frac{1}{4} \text{ of prob. contained in this lobe} \]

1.d) \( \int_{-\infty}^{\infty} \Psi_{1} \Psi_{2} \, dx = \frac{2}{L} \int_{0}^{L} \sin \left( \frac{\pi x}{L} \right) \sin \left( \frac{2\pi x}{L} \right) \, dx \)

\[ \sin (2y) = 2 \sin y \cos y \]

\[ = \frac{4}{L} \int_{0}^{L} \sin^2 \left( \frac{\pi x}{L} \right) \cos \left( \frac{\pi x}{L} \right) \, dx \]

let \( u = \sin \left( \frac{\pi x}{L} \right) ; \, du = \frac{\pi}{L} \cos \left( \frac{\pi x}{L} \right) \, dx \)

\[ = \frac{4}{L} \left( \frac{1}{\pi} \right) \int_{0}^{\sin (1)} u^2 \, du = \frac{4}{\pi} \left[ \frac{1}{3} u^3 \right]_{0}^{1} = 0 \quad \therefore \]
l.e) P.B. w.F. \( \psi_n = \sqrt{\frac{2}{L}} \sin \left( \frac{n\pi x}{L} \right) \)

Energy \( E = \frac{n^2 \hbar^2}{8mL^2} \)

as \( L \to \infty \),

1) the spacing between energy levels \( \to 0 \)

\( \Delta E_{n,n-1} = (2n-1) \frac{\hbar^2}{8mL^2} \)

thus, quantization of energy is lost - like the Free particle.

2) For a state with finite energy, \( n \to \infty \) and \( |\psi|^2 \) approaches a uniform function with an infinite number of nodes that merge together into a continuum-like the Free particle.

3) Z.P. E. \( = \frac{\hbar^2}{8mL^2} \to 0 \), thus the state with momentum \( \to 0 \) becomes allowed-like the Free particle.
2.a) Z.P.E., where \( n_x = n_y = 1 \)

\[
Z \text{P.E.} = \left( \frac{1}{L^2} + \frac{1}{(2L)^2} \right) \frac{\hbar^2}{8m} = \frac{5}{4} \frac{\hbar^2}{8mL^2} = \frac{5\hbar^2}{32L^2}
\]

2.b) \text{degeneracy occurs when two states have the same energy.}
\text{states correspond to different sets of } n_x \text{ and } n_y

\[ E = \left( \frac{n_x^2}{L^2} + \frac{n_y^2}{4L^2} \right) \frac{\hbar^2}{8m} \]

\text{degeneracy occurs when}

\[
\frac{n_x^2}{L^2} + \frac{n_y^2}{4L^2} = \frac{n_x'^2}{L^2} + \frac{n_y'^2}{4L^2}
\]

\[ 4n_x^2 + n_y^2 = 4n_x'^2 + n_y'^2 \]

\text{occurs when}

\[ n_x = 1, \ n_y = 4 \ \text{and} \ n_x' = 2, \ n_y' = 2 \]

\[ 4 + 16 = 16 + 4 \]

or when

\[ n_x = 1, \ n_y = 8 \ \text{and} \ n_x' = 4, \ n_y' = 2 \]

\[ 4 + 64 = 64 + 4 \]

\text{just need to identify 1 pair of states.}

\text{note: these are easy to identify graphically.}
2. (c) i) \( \Psi_{n_{x}=2, n_{y}=4} = \frac{\sqrt{2}}{L_{x}} \frac{\sqrt{2}}{L_{y}} \sin\left(\frac{n_{x} \pi x}{L_{x}}\right) \sin\left(\frac{n_{y} \pi y}{L_{y}}\right) \)

\[ = \frac{\sqrt{2}}{L} \sin\left(\frac{2 \pi x}{L}\right) \sin\left(\frac{2 \pi y}{L}\right) \]

3.a) \( V \)

\( V \)

\( \Psi \)

\( V \)

\( -\infty \quad 0 \quad L \quad \infty \)

\( \text{region: I} \quad \text{II} \quad \text{III} \)

\( \Psi_{\text{III}} = D e^{-\frac{\sqrt{2m(V-E)}}{\hbar} (x-L)} \)

when \( V = E \), exponent = 0

\( \Psi_{\text{III}} = D e^{0} \) in region of well

penetration depth defined as value of \( x \) where the value of \( \Psi \) is \( \frac{1}{e} \) its value at \( x = L \).
ψ_{II} is constant for all \( x > L \), so it never decays. penetration depth is essentially infinite!

3.b) Energy expression for P. B. with one finite wall:

From lecture notes,

\[
-\tan \left( \frac{\sqrt{2mE}}{\hbar} \frac{L}{L} \right) = \frac{\sqrt{E}}{\sqrt{V-E}}
\]

As \( V \to \infty \), \( \frac{\sqrt{E}}{\sqrt{V-E}} \to 0 \).

so \(-\tan \left( \frac{\sqrt{2mE}}{\hbar} \frac{L}{L} \right) = 0 \) true when argument = \( n \pi \)

\( n = 1, 2, 3, \ldots \)

exclude \( n = 0 \).

\[
\frac{\sqrt{2mE}}{\hbar} = n\pi
\]

\[
\frac{2\sqrt{2mE}L}{\hbar} = n
\]

\[
E = \frac{n^2\hbar^2}{8mL^2}
\]