Name: ____________________________

Lecture Problem Set #5

Due Monday, Oct 4, 2010

(10 points)

1. The following questions regard the quantum mechanical particle on a sphere.
   a) Is the energy, \( E = 2.695 \times 10^{-20} \text{ J} \), an allowed energy level for a particle on a sphere with a moment of inertia \( I = 1.448 \times 10^{-46} \text{ kgm}^2 \)? If so, calculate the angular momentum and degeneracy of this state. If not, explain why it is not allowed.
   b) The spherical harmonic solution that we will associate with a \( p_z \) orbital of the hydrogen atom is \( Y_{1,0}(\theta, \phi) = N_{1,0} \cos \theta \). The other two solutions corresponding to \( l = 1 \) are \( Y_{1,\pm 1}(\theta, \phi) = m V_{1,1} \sin \theta e^{\pm i \phi} \). Use these solutions to construct functions that we will associate with \( p_x \) and \( p_y \) orbitals of the Hydrogen atom. *Hint:* recall the relationships between complex exponentials and the sine and cosine functions (see p. 286 of Atkins).
   c) Atkins: Problem 8.29. Show this using spherical polar coordinates. *Hint:* remember that the order of partial derivatives is not important.
   d) Calculate the angular momentum quantum number for a gyroscope with moment of inertia \( I = 1.80 \times 10^{-4} \text{ kgm}^2 \) spinning with angular velocity \( \omega = 314 \text{ rad/s} \). Recall that the classical angular momentum is given by \( I\omega \).
   e) At what angular velocity would the gyroscope spin if it resided in the \( l = 6 \) state of the particle on a sphere? Would this motion be perceptible?

2. The following questions regard hydrogenic atoms.
   a) Calculate the ionization energy of a \( \text{Li}^{2+} \) ion in the state with \( n = 3 \). Give your answer in electron volts. Compare this value to the ionization energy of a \( \text{H} \) atom in the state \( n = 1 \).
   b) In the derivation of the radial portion of the Schrödinger equation of hydrogenic atoms, we encountered two potential energy terms that we grouped together as \( V' \). Write the explicit expression for \( V' \) and describe the physical meaning of the terms.
   c) Write the complete expressions for the following wavefunctions: \( \psi_{1,0,0} \); \( \psi_{2,1,0} \); \( \psi_{3,2,0} \).
   d) Graph the radial components of the wavefunctions you wrote in part c).
   e) Graph the angular components of the wavefunctions you wrote in part c).