Homework Assignment #2

Atkins

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Lowe’s Problems from chapter B2

1) Exercise 15:
   a. Show that, when V is a constant, \( \exp(\pm i \sqrt{2m(E-V)} x / \hbar) \) is an eigenfunction for the Schrödinger equation in one dimension, with eigenvalue E.
   b. Show that, when \( V > E \), the functions of part a. become exponentials with real arguments.
   c. What is the difference between the two exponential functions of part b? Can either of them be discarded on the basis of being unacceptable as a wavefunction if the range of x being considered is \( L \leq x \leq \infty \)?
   d. How is the behavior of the remaining exponential influenced by the magnitude of \( E - V \)? What happens as \( V \) approaches infinity?

2) Problem 5: A particle of mass m is located in a 1-dimensional box with walls at x = 0 and x = L. For the fourth (n = 4) energy level,
   a. Set up the integral necessary to determine the probability of finding the particle between x = 0.0 and x = 0.1L.
   b. Make a rough estimate of the value of the integral.
   c. Evaluate the integral exactly.

3) Problem 6: We usually define the box as lying in the range x = 0 to x = L. The solutions are \( \psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \). What would the solutions be if the box were defined to be in the range x = -L/2 to x = L/2? (You should be able to answer this by inspection and symmetry arguments.)

(continued on next page)
4) This problem is related to the particle in a box with one finite wall.

The wavefunction of the particle is \( \psi(x) = \begin{cases} 
0 & \text{if } x \geq L \\
B \sin \left( \frac{\sqrt{2mE} x}{\hbar} \right) & \text{if } 0 < x < L \\
D \exp \left( - \frac{\sqrt{2m(V - E)} x}{\hbar} \right) & \text{if } x \geq L
\end{cases} \)

(The constants \( B \) and \( D \) are related to each other, but their values are not important here.)

a. Sketch the ground state solution of the particle in a box with one finite wall on the graph shown below. Use your pencil to shade the region of the wavefunction that is classically forbidden.

b. The allowed energies of the particle must satisfy the equation

\[
- \tan \left( \frac{\sqrt{2mE} L}{\hbar} \right) = \frac{\sqrt{E}}{\sqrt{V - E}}
\]

Show that the allowed energies approach those of the particle in a box with infinite walls as the potential, \( V \), approaches infinity.

c. Describe how the penetration depth increases with increasing mass and decreasing energy relative to the potential.